Homework 6 of Optimization-2024"

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Requirement: Please write the answers **in English**.

Reference Textbook: [1]"Stephen Boyd and Lieven Vandenberghe, "Convex optimization", 2th Edition, 2013.

[2] 刘浩洋, 户将, 李勇锋, 文再文, 最优化: 建模, 算法与理论, 高等教育出版社,2020.

Refer Books: [3] Simon Foucart, Holger Rauhut, A Mathematical Introduction to Compressive Sensing, Springer, 2013.

1. (40 points)

(i) Please solve the proximal of ℓ_∞ norm $f(\mathbf{x}) = \|\mathbf{x}\|_1 = \sum_{j=1}^n |x_j|$. Tips: You can solve it by Moreau decomposition and the projection on the ℓ_1 ball.

(ii) (Example 8.1 of [2])Please compute the proximal operator of $f(\mathbf{x}) = -\sum_{j=1}^n \ln x_j$.

(iii) (习题8.4 of [2])Please compute the proximal operator of $f(\mathbf{X}) = -\ln\det(\mathbf{X})$, where $\mathrm{dom}(f) = \{\mathbf{X} : \mathbf{X} \succ \mathbf{O}\}$.

2. (30 points)Please solve the following low-rank matrix recovery problem via Proximal Gradient descent

$$\min_{\mathbf{X}\in\mathbb{R}^{n_1\times n_2}} \lambda\|\mathbf{X}\|_* + \frac{1}{2}\|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_2^2$$

where $\mathbf{b} = \mathcal{A}(\mathbf{X}_0) = (\langle \mathbf{A}_j, \mathbf{X}_0\rangle)_{j=1}^m$ with $\mathbf{A}_j \in \mathbb{R}^{n_1\times n_2}$ with $m = \mathcal{O}((n_1+n_2)r)$, and $\|\cdot\|_*$ is the nuclear norm.

3. (30 points)Given the convex problem $\min_{\mathbf{x}} \|\mathbf{x}\|_1$, s. t. $\|\mathbf{A}^*(\mathbf{A}\mathbf{x} - \mathbf{b})\|_\infty \le \varepsilon$, where $\mathbf{A}^* \in \mathbb{R}^{n\times m}$ is the transpose of the matrix $\mathbf{A} \in \mathbb{R}^{m\times n}$ with $m < n$. Please design an solving algorithm based on dual algorithm. You should give the iterated scheme in detail.

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