Homework 5 of Optimization-2024"

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Requirement: Please write the answers **in English**.

Reference Textbook: [1]"Stephen Boyd and Lieven Vandenberghe, "Convex optimization", 2th Edition, 2013.

[2] 刘浩洋, 户将, 李勇锋, 文再文, 最优化: 建模, 算法与理论, 高等教育出版社,2020.

1. (30 points)Please solve the following low-rank matrix recovery problem via Project Gradient descent

$$\min_{\text{rank}(\mathbf{X})\leq r} \frac{1}{2}\|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_2^2$$

where $\mathbf{b} = \mathcal{A}(\mathbf{X}_0) = (\langle \mathbf{A}_j, \mathbf{X}_0\rangle)_{j=1}^m$ with $\mathbf{A}_j \in \mathbb{R}^{n_1\times n_2}$ with $m = \mathcal{O}((n_1+n_2)r)$. Please refer to the following paper.

[Jain P, Meka R, Dhillon I. Guaranteed rank minimization via singular value projection[J]. Advances in Neural Information Processing Systems, 2010, 23.]

2. (30 points)(Exercises 2.13 of the textbook [2])Please give the statement of the subgradient of the following functions:

(a) $f(\mathbf{x}) = \|\mathbf{A}\mathbf{x} - b\|_2 + \|\mathbf{x}\|_2$;

(b) $f(\mathbf{x}) = \inf_{\mathbf{y}} \|\mathbf{A}\mathbf{y} - \mathbf{x}\|_\infty$, 这里可以假设能够取到$\hat{\mathbf{y}}$, 使得$\|\mathbf{A}\hat{\mathbf{y}} - \mathbf{x}\|_\infty = f(\mathbf{x})$. (Please refer to Example 2.16 of the textbook [2])

3. (40 points)Please solve the real phase retrieval problem via Newton Method. Consider the problem

$$\min_{\mathbf{x}\in\mathbb{R}^n} f(\mathbf{x}) := \frac{1}{2m}\sum_{j=1}^m \left(\langle \mathbf{a}_j, \mathbf{x}\rangle^2 - b_j\right)^2,$$

where $b_j = \langle \mathbf{a}_j, \mathbf{x}_0\rangle^2$ and $\mathbf{a}_j \in \mathbb{R}^n,\ j = 1,\ldots,m$ are Gaussian random measurements with $m \geq \mathcal{O}(n\log n)$. Please refer to the following paper.

[Gao Bing, Xu Zhiqiang. Phaseless recovery using the Gauss – Newton method. IEEE Transactions on Signal Processing, 2017, 65(22): 5885-5896.]

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