

## Homework 9 of Optimization-2024”

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Requirement: Please write the answers **in English**.

1. (40 points) Consider the **sparse phase retrieval** problem  $\mathbf{b} = |\mathbf{A}\mathbf{x}_0|^2 + \mathbf{e} \in \mathbb{R}^m$ . We can solve it via the following model

$$\min_{\mathbf{X} \succeq \mathbf{O}} \frac{\|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_2^2/2 + \lambda \text{Tr}(\mathbf{X}) + \mu \|\mathbf{X}\|_1}{\|\mathbf{X}\|_F},$$

where  $\mathcal{A} : \mathbb{C}^{n \times n} \rightarrow \mathbb{R}^m$ ,  $\mathcal{A}(\mathbf{X})_j = \langle \mathbf{a}_j \mathbf{a}_j^*, \mathbf{X} \rangle =: \langle \mathbf{A}_j, \mathbf{X} \rangle$ . Please design an solving algorithm and give the iterated scheme.

[Xia Y, Xu Z. Sparse phase retrieval via phaselift-off. IEEE Transactions on Signal Processing, 2021, 69: 2129-2143.]

2. (60 points) Please give the projector operator, Tangent space, normal space, Riemannian gradient, Riemannian Hessian of the following Grassmann manifold

$$\text{Grass}(n, p) = \{\text{Span}(\mathbf{X}) : \mathbf{X} \in \mathbb{R}^{n \times p}, \mathbf{X}^\top \mathbf{X} = \mathbf{I}_p\} = \text{St}(n, p) / \mathcal{O}(p)$$

where  $\mathcal{O}(p) = \{\mathbf{X} \in \mathbb{R}^{p \times p} : \mathbf{X}^\top \mathbf{X} = \mathbf{I}_p\}$  is the orthogonal group, and  $\text{St}(n, p) = \{\mathbf{X} \in \mathbb{R}^{n \times p} : \mathbf{X}^\top \mathbf{X} = \mathbf{I}_p\}$  is the Stiefel orthogonal group. Please give the details rather than only the final results. Tips: You can refer to Section 9.16 of [Boumal N. An introduction to optimization on smooth manifolds[M]. Cambridge University Press, 2023.]