Homework 8 of Optimization-2024"

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Requirement: Please write the answers **in English**.

1. (50 points) Consider the **low-rank matrix recovery** problem $\mathbf{b} = \mathcal{A}(\mathbf{X}_0) + \mathbf{e} \in \mathbb{R}^m$. We can solve it via the following matrix decomposition model

$$\min_{\mathbf{U} \in \mathbb{R}^{n_1 \times r}, \mathbf{V} \in \mathbb{R}^{n_2 \times r}} \frac{1}{2} \|\mathcal{A}(\mathbf{U}\mathbf{V}^*) - \mathbf{b}\|_2^2 + \lambda \|\mathbf{U}^*\mathbf{U} - \mathbf{V}^*\mathbf{V}\|_F^2,$$

where $\mathcal{A} : \mathbb{C}^{n_1 \times n_2} \to \mathbb{R}^m, \mathcal{A}(\mathbf{X})_j = \langle \mathbf{A}_j, \mathbf{X} \rangle$ with $\mathbf{A}_j \in \mathbb{R}^{n_1 \times n_2}$. Please design an solving algorithm via BCD and give the iterated scheme. Tips: You can solve each subproblems via Gradient Descent.

2. (50 points) Consider the **sparse phase retrieval** problem $\mathbf{b} = |\mathbf{A}\mathbf{x}_0|^2 + \mathbf{e} \in \mathbb{R}^m$. We can solve it via the following model

$$\min_{\mathbf{x}, \mathbf{y} \in \mathbb{C}^n} \frac{1}{2} \|\mathcal{A}(\mathbf{x}\mathbf{y}^*) - \mathbf{b}\|_2^2 + \lambda \|\mathbf{x} - \mathbf{y}\|_2^2 + \rho \|\mathbf{x}\|_1 + \delta \|\mathbf{y}\|_1,$$

where $\mathcal{A} : \mathbb{C}^{n \times n} \to \mathbb{R}^m, \mathcal{A}(\mathbf{X})_j = \langle \mathbf{a}_j \mathbf{a}_j^*, \mathbf{X} \rangle =: \langle \mathbf{A}_j, \mathbf{X} \rangle$. Please design an solving algorithm via BCD and give the iterated scheme. Tips: You can solve each subproblems via Proximal Gradient Descent.

[Cai J F, Liu H, Wang Y. Fast rank-one alternating minimization algorithm for phase retrieval[J]. Journal of Scientific Computing, 2019, 79: 128-147.]

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