Homework 7-2023"

Peng Li *

May 13, 2024

Requirement: Please write the answers **in English**.

Reference Papers:

[1]Wang, Lie. " The L1 penalized LAD estimator for high dimensional linear regression." Journal of Multivariate Analysis 120 (2013): 135-151.

[2] Yang, Junfeng, and Yin Zhang. "Alternating direction algorithms for ℓ_1-problems in compressive sensing." SIAM journal on scientific computing 33.1 (2011): 250-278.

[3] Wen, Fei, et al. "Robust Sparse Recovery in Impulsive Noise via $\ell_p - \ell_1$ Optimization." IEEE Transactions on Signal Processing 65.1 (2016): 105-118.

1. (50 points) Low-rank Matrix Recovery. Given the observation $\mathbf{b} = \mathcal{A}(\mathbf{X}_0) + \mathbf{e} \in \mathbb{R}^m$, where the mapping $\mathcal{A}(\mathbf{X}) = (\langle A_j, \mathbf{X} \rangle)_{j=1}^m$ for any matrix $\mathbf{X} \in \mathbb{R}^{n_1 \times n_2}$ and $m < \min\{n_1, n_2\}$, $\mathbf{e} \in \mathbb{R}^m$ is the possible error or noise. We aim to recover the unknown low-rank matrix $\mathbf{X}_0$ from the observations $\mathbf{b}$. Consider the matrix regularized least absolute deviation (Matrix RLAD) model

$$\min_{\mathbf{X}} \|\mathcal{A}(\mathbf{X}) - \mathbf{b}\|_1 + \lambda \|\mathbf{X}\|_*.$$

where $\|\cdot\|_*$ is the nuclear norm and $\|\cdot\|_1$ is the ℓ_1 norm. Please design an solving algorithm based on ADMM and give the closed solution of each subproblem.

2. (50 points) Robust PCA. Given the observation $\mathbf{b} = \mathcal{A}(\mathbf{L}_0 + \mathbf{S}_0) + \mathbf{e} \in \mathbb{R}^m$, where the mapping $\mathcal{A}(\mathbf{X}) = (\langle \mathbf{A}_j, \mathbf{X}_0 \rangle)_{j=1}^m$ for any $\mathbf{X} \in \mathbb{R}^{n_1 \times n_2}$ and $m < \min\{n_1, n_2\}$, $\mathbf{e} \in \mathbb{R}^m$ is the possible error or noise. We aim to recover the unknown low-rank matrix $\mathbf{L}_0$ and sparse matrix $\mathbf{S}_0$ from the observations $\mathbf{b}$. Consider the Matrix Lasso model with two variables

$$\min_{\mathbf{L},\mathbf{S}} \frac{1}{2\rho} \|\mathcal{A}(\mathbf{L} + \mathbf{S}) - \mathbf{b}\|_2^2 + \lambda \|\mathbf{L}\|_* + \|\mathbf{S}\|_1.$$

where $\|\cdot\|_*$ is the nuclear norm and $\|\cdot\|_1$ is the ℓ_1 norm. Please design an solving algorithm based on ADMM and give the closed solution of each subproblem.

Email:lp@lzu.edu.cn.