# Homework 3 (4th Week)-2023”

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Requirement: Please write the answers **in English**.

Reference Textbook: [1]“Stephen Boyd and Lieven Vandenberghe, “Convex optimization”, 2th Edition, 2013.

[2] 刘浩洋, 户将, 李勇锋, 文再文, 最优化: 建模, 算法与理论, 高等教育出版社,2020.

1. (30 points) (Exercise 4.24 of the textbook [1]) Complex $\ell_1-, \ell_2-$ and $\ell_\infty$-norm approximation. Consider the problem

$$\text{minimize } \|\mathbf{Ax} - \mathbf{b}\|_p,$$

where $\mathbf{A} \in \mathbf{C}^{m \times n}, \mathbf{b} \in \mathbf{C}^m$, and the variable is $\mathbf{x} \in \mathbf{C}^n$. The complex $\ell_p$-norm is defined by

$$\|\mathbf{y}\|_p = \left( \sum_{i=1}^m |y_i|^p \right)^{1/p}$$

for $p \geq 1$, and $\|\mathbf{y}\|_\infty = \max_{i=1,\ldots,m} |y_i|$. For $p = 1, 2$, and $\infty$, express the complex $\ell_p$-norm approximation problem as a QCQP or SOCP with real variables and data.

2.(30 points) (Exercise 5.30 of the textbook [1]) Derive the KKT conditions for the problem

$$\begin{array}{ll} \text{minimize} & \operatorname{tr}(\mathbf{X}) - \log\det\mathbf{X} \\ \text{subject to} & \mathbf{Xs} = \mathbf{y}, \end{array}$$

with variable $\mathbf{X} \in \mathbf{S}^n$ and domain $\mathbf{S}^n_{++}$. Here $\mathbf{y} \in \mathbf{R}^n$ and $\mathbf{s} \in \mathbf{R}^n$ are given, with $\mathbf{s}^T\mathbf{y} = 1$. Verify that the optimal solution is given by

$$\mathbf{X}^\star = \mathbf{I} + \mathbf{y}\mathbf{y}^T - \frac{1}{\mathbf{s}^T\mathbf{s}}\mathbf{s}\mathbf{s}^T.$$

3. (40 points)Given $\mathbf{A} \in \mathbb{R}^{m \times n}$ with $m < n$ and $\mathbf{b} \in \mathbb{R}^m$. Please give the dual problems of the following convex optimizations.

(i) $\min_{\mathbf{x} \in \mathbb{R}^n} \|\mathbf{x}\|_1,\ s.\ t.\ \|\mathbf{A}^\top(\mathbf{Ax} - \mathbf{b})\|_\infty \leq \varepsilon$

(ii) $\min_{\mathbf{x} \in \mathbb{R}^n} \lambda\|\mathbf{x}\|_1 + \|\mathbf{A}^\top(\mathbf{Ax} - \mathbf{b})\|_\infty$

(iii) $\min_{\mathbf{x} \in \mathbb{R}^n} \lambda\|\mathbf{x}\|_1 + \frac{1}{2}\|\mathbf{A}^\top(\mathbf{Ax} - \mathbf{b})\|_\infty^2$

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