Homework 2 of Optimization (4th Week)-2024"

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Requirement: Please write the answers **in English**.

Reference Textbook: [1]"Stephen Boyd and Lieven Vandenberghe, "Convex optimization", 2th Edition, 2013.

[2] 刘浩洋, 户将, 李勇锋, 文再文, 最优化: 建模, 算法与理论, 高等教育出版社,2020.

1. (30 points) (Exercises 3.18, 3.57 of the textbook [1])

(a) $f(\mathbf{X}) = \operatorname{tr}\left(\mathbf{X}^{-1}\right)$ is convex on $\operatorname{dom} f = \mathrm{S}_{++}^n$.

(b) $f(\mathbf{X}) = (\det(\mathbf{X}))^{1/n}$ is concave on $\operatorname{dom} f = \mathrm{S}_{++}^n$.

2. (30 points) (Exercise 2.12 of the textbook [2], Exercises 3.36 of the textbook [1]) Derive the conjugates of the following functions.

(a) Max function. $f(\mathbf{x}) = \max_{i=1,\ldots,n} x_i$ on $\mathbf{R}^n$.

(b) Sum of largest elements. $f(\mathbf{x}) = \sum_{i=1}^r x_{[i]}$ on $\mathbf{R}^n$.

(c) Log function of the Matrix: $f(\mathbf{X}) = -\ln\det(\mathbf{X})$;

3. (40 points)(Exercise 2.6 of the textbook [2])Compute the gradient of the functions with matrix variables.

(a) $f(\mathbf{X}) = \operatorname{Tr}\left(\mathbf{X}^{\mathrm{T}}\mathbf{A}\mathbf{X}\right)$, where $\mathbf{X} \in \mathbb{R}^{m\times n}$, $\mathbf{A} \in \mathbb{R}^{m\times m}$(may not symmetrix);

(b) $f(\mathbf{X}) = \ln\det(\mathbf{X})$, where $\mathbf{X} \in \mathbb{R}^{n\times n}$ and domain is $\{\mathbf{X} \mid \det(\mathbf{X}) > 0\}$.

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